

Isoelastic demand:

$$\varepsilon = -1$$

$$\frac{dQ}{dP} \frac{P}{Q} = -1$$

$$\frac{1}{Q} dQ = -\frac{1}{P} dP$$

$$\int \frac{1}{Q} dQ = -\int \frac{1}{P} dP$$

$$\ln Q + C_1 = -\ln P + C_2$$

$$\ln Q = \ln P^{-1} + C_2 - C_1$$

$$e^{\ln Q} = e^{\ln P^{-1} + C_2 - C_1}$$

$$Q = e^{\ln P^{-1}} e^{C_2 - C_1}$$

$$Q = P^{-1} e^{C_2 - C_1}$$

$$QP = e^{C_2 - C_1}$$

$$= k$$

So total expenditure  $P \cdot Q$  does not change along the demand curve.