

Signs of Derivatives (Version 3)

Basic Definitions and Relationships

1. Marginal Utility of $x \equiv \frac{\partial U}{\partial x}$

2. $MRS \equiv \frac{MU_x}{MU_y}$

3. Slope of Indifference Curve $\equiv -MRS$

4. $\frac{\partial(\text{slope of IC})}{\partial x} \equiv -\frac{\partial MRS}{\partial x}$

More is Better

Holding y constant, x has the property of:

- i. More is Better (good)

$$\frac{\partial U}{\partial x} > 0$$

- ii. More does not matter (neutral good)

$$\frac{\partial U}{\partial x} = 0$$

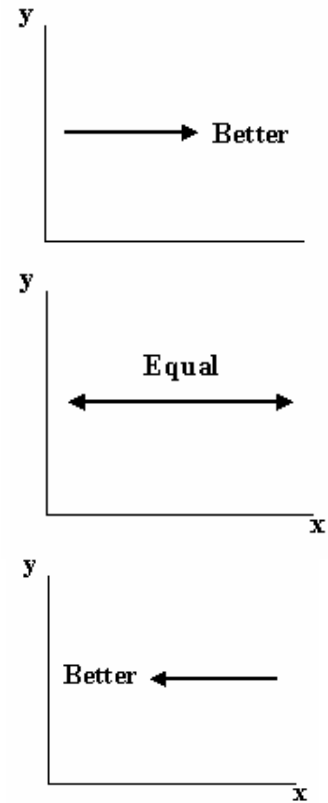
- iii. More is Worse (bad)

$$\frac{\partial U}{\partial x} < 0$$

Remember that since y is on the vertical axis the arrows for y are 90 degrees anti-clockwise to that of x .

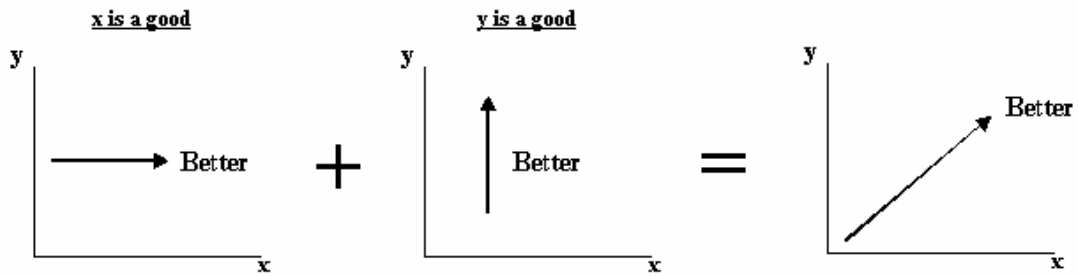
Knowing the above for both x and y you can solve for

- i. Towards which direction of the graph is better
ii. Slope of IC



Direction of Graph

e.g. x and y are both goods:



Slope of IC

Slope of Indifference Curve $\equiv -MRS \equiv -\frac{MU_x}{MU_y}$

So if

x	y	MRS	IC slope	Possible Graph(s)
good	good	+	-	
good	bad	-	+	
bad	good	-	+	
good	neutral	∞	$-\infty$	
neutral	good	0	0	

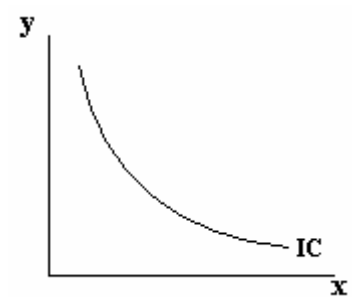
Marginal Rate of Substitution

A. If x and y are both goods,

i. Diminishing Marginal Rate of Substitution

$$\frac{\partial MRS}{\partial x} < 0$$

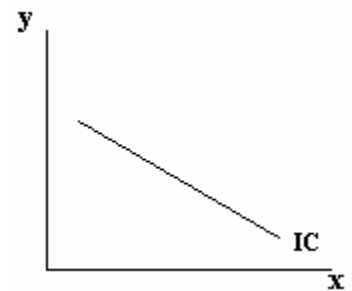
- Prefer average bundles
- Interior solution if slope of BC = slope of IC at some positive x and y; corner solution otherwise
- Convex IC



ii. Constant Marginal Rate of Substitution

$$\frac{\partial MRS}{\partial x} = 0$$

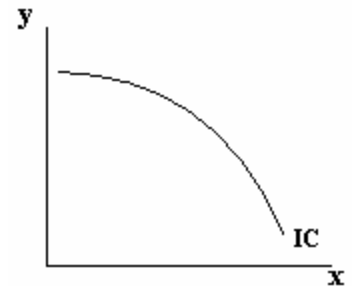
- Perfect Substitutes
- Whole BC is solution if slope of BC = slope of IC; corner solution otherwise
- Straight line IC



iii. Increasing Marginal Rate of Substitution

$$\frac{\partial MRS}{\partial x} > 0$$

- Prefer Extremes—corner solution
- Concave IC



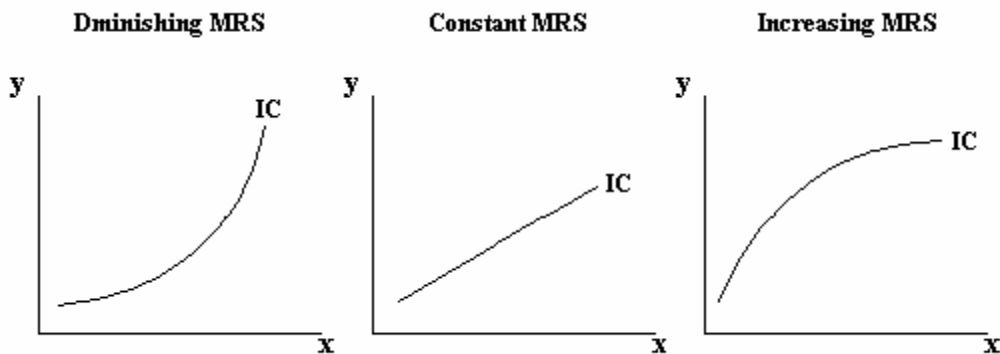
B. If x is a good and y is a bad/neutral

Corner solution—consume only x

C. If x is a bad/neutral and y is a good

Corner solution—consume only y

Graphs for one good and one bad

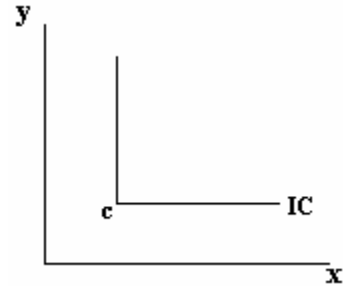


Special Cases

$\min\{A,B\}$ - Perfect Complements

Perfect complements has “L” shaped IC’s. The kink c always satisfies $A = B$.

Optimal Solution: Always at the kink. Find the relationship between x and y with $A = B$, substitute that into the budget constraint and solve for x and y .



MRS: MRS is 0 at the horizontal portion of an indifference curve and infinite at the vertical portion.

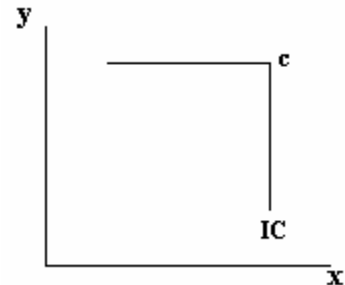
Slope of IC: Slope of IC is 0 at the horizontal portion of an indifference curve and negative infinite at the vertical portion.

$\max\{A,B\}$

The kink c always satisfies $A = B$.

Optimal Solution: Always corner solution.

MRS: MRS is 0 at the horizontal portion of an indifference curve and infinite at the vertical portion.



Slope of IC: Slope of IC is 0 at the horizontal portion of an indifference curve and negative infinite at the vertical portion.