University of California, Berkeley Fall 2006
ECON 100A Section 111, 112

10th Section Production (Version 2)

Market is composed of demand and supply. For the past few weeks we have focused on demand, developing theories on consumer choice. We shall now turn to the other side of the market—supply, which is the behavior of firms (producers).

I. Analogy to Consumer Choice

By now you should be familiar with consumer choice theory. If so, congratulations! The theory of producer choice has many things in common with the theory of consumer choice. The list below shows the analogy:

Analogy between Producer Choice Theory and Consumer Choice Theory

Consumer Choice	Producer Choice
Utility (Function)	Production (Function)
Prices of Goods	Input Prices
Indifference Curve	Isoquant
Marginal Utility	Marginal Product
Marginal Rate of Substitution (MRS)	Marginal Rate of Technical Substitution (MRTS)
Diminishing Marginal Utility	Diminishing Marginal Returns
Diminishing MRS	Diminishing MRTS

There are some differences though. First, producers are not bound by a fixed income, so there is no budget constraint. Second, producers do not maximize production; rather, they maximize profit, which depends on the price of their output.

II. Short Run and Long Run

Short Run—Some Variables are unchangeable

Long Run—All Variables are changeable

Short run (SR) and long run (LR) has no absolute relation with time. The two concepts are defined as a description of whether all variables are changeable or not. The best way to think of the two concepts is to take short run as sudden change and long run as planned change.

In this class we almost always assume firms use only two inputs—labor (L) and capital (K). Take anything that is not labor as capital for the time being—machines, factories, etc. We usually assume K to be fixed and L variable in the short run.

Examples of Short Run:

-A factory decides to add another shift

III. Average Product and Marginal Product

Let Q be output quantity and X be the quantity of an input,

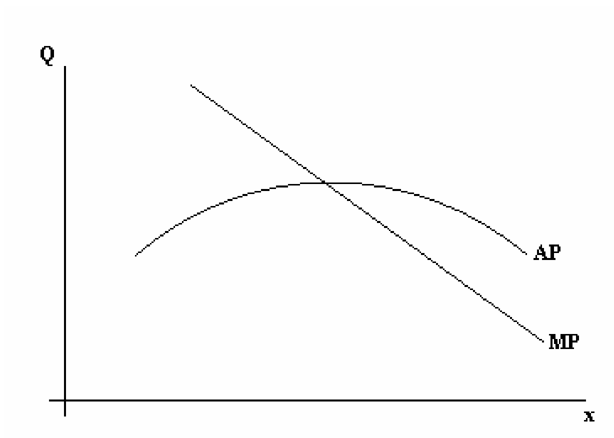
Average Product : $AP = \frac{Q}{X}$ *How many units is each worker producing on average*

Marginal Product : $MP = \frac{\partial Q}{\partial X}$ *How many units is the last worker producing*

<u>Diminishing Marginal Returns</u>

Holding other inputs constant,

$$\frac{\partial MP}{\partial X} < 0$$



DMR applies only to short run since other inputs are held constant.

Note that if MP is diminishing, the maximum point of AP corresponds to the intersection between AP and MP.

IV. Economies of Scale

When input increase by c %,

If there is…	Then Output…
Increasing Returns to Scale	Increase more than c %
Constant Returns to Scale	Increase than c %
Decreasing Returns to Scale	Increase less than c %

This is equivalent to

If there is…	Then $F(K,L)$ satisfies…
Increasing Returns to Scale	$F(\alpha \cdot K, \alpha \cdot L) > \alpha \cdot F(K,L)$
Constant Returns to Scale	$F(\alpha \cdot K, \alpha \cdot L) = \alpha \cdot F(K,L)$
Decreasing Returns to Scale	$F(\alpha \cdot K, \alpha \cdot L) < \alpha \cdot F(K,L)$

Since α represents a percentage increase, it is <u>always bigger than 1</u> when we talk about Returns to Scale.

Economies of Scale are about long run—all factors are changeable. A common mistake is to confuse diseconomies of scale with diminishing marginal return. The former happens in the long run while the later only happens in the short run.

In addition, returns to scale can change across different combination of inputs—a production function can exhibit increasing returns to scale for some values of K and L while having decreasing returns to scale for some others.

Find the returns to scale of the following:

i. $F(K,L) = AL^{1/2}K^{1/2}$

$$\begin{aligned} F(\alpha K, \alpha L) &= A(\alpha L)^{1/2}(\alpha K)^{1/2} \\ &= A\alpha^{1/2}L^{1/2}\alpha^{1/2}K^{1/2} \\ &= \alpha A L^{1/2}K^{1/2} \\ &= \alpha F(K,L) \end{aligned}$$

so <u>CRS</u>

ii. $F(K,L) = 2L + LK + K$

$$\begin{aligned} F(\alpha K, \alpha L) &= 2\alpha L + \alpha L \alpha K + \alpha K \\ &= \alpha 2(L + \alpha LK + K) \\ &> \alpha F(K,L) \end{aligned}$$

so <u>IRS</u>

iii. $F(K,L) = L^a K^{1-a}, a \in (0,1)$

$$\begin{aligned} F(\alpha K, \alpha L) &= A(\alpha L)^{a}(\alpha K)^{1-a} \\ &= A\alpha^{a}L^{a}\alpha^{1-a}K^{1-a} \\ &= \alpha A L^{a}K^{1-a} \\ &= \alpha F(K,L) \end{aligned}$$

so <u>CRS</u>

Note that example i. is a special case of example iii. Such production functions are called *Cobb-Douglas production functions*. Note that the CRS result depends on the power coefficients (α and 1-α) summing up to one. Sometimes people call a function Cobb-Douglas even if the coefficients do not sum up to one; in that case the function is not necessarily CRS.

V. Example

1. Assume the only variable input in the production of computers is Labor. Complete the following chart.

Quantity of Labor	Total Output	Marginal Product of Labor	Average Product of Labor
0	0	-	-
1		200	
2			175
3	450		
4		70	
5			115
6		43	

2. For each of the following production functions: (1) state whether it has increasing, decreasing, or constant returns to scale, and (2) find the MRTS of L for K.

   (a) $q(K, L) = AL^{1/2}K^{1/2}$

   (b) $q(K, L) = 2L + LK + K$

   (c) $q(K, L) = L^{1/3}K^{1/3}$

   (d) $q(K, L) = 6L^{2/3}K^{5/6}$

   (e) $q(K, L) = L^{\alpha}K^{1-\alpha}, \quad \alpha \in (0, 1)$