University of California, Berkeley Fall 2006

ECON 100A Section 111, 112

12th Section Product Maximization and Cost Minimization

Recall that in consumer choice we take budget constraint as fixed and move indifference curves to find the optimal point. The analogy of firm/producer/seller choice is a bit different, since a firm is not bounded by a fixed income. The optimization could go in two directions—either we maximize production for a given expenditure amount (cost), or we minimize cost for a given production quantity. In this handout we show how to proceed with both and show that they give us the same optimal production—cost combination.

I. Setting

Variables: K, L

Production function $F(K,L)$ given.

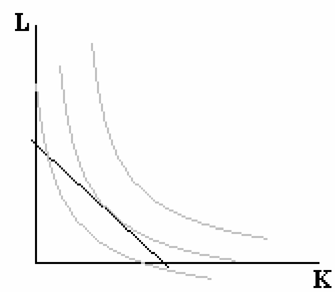
Prices for K and L—r and w—are fixed. Total cost is $rK + wL$.

$\overline{F}$, $\overline{C}$ are constants.

i. Product Maximization

$$\max\{F(K,L)\} \qquad \text{s.t.} \quad rK + wL = \overline{C}$$

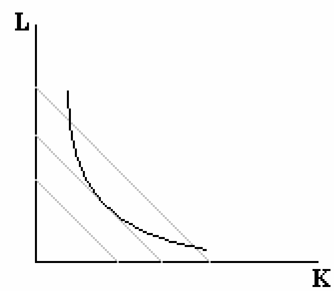
Production maximization is a direct analogy to utility maximization—we literally work through the same math, just with different notations.



ii. Cost Minimization

$$\min\{rK + wL\} \qquad \text{s.t.} \quad F(K,L) = \overline{F}$$

In cost minimization we are doing the reverse; we move the "budget constraint" until we find the optimum.



II. Equaling of Slopes

By equaling the slopes of the isoquant and isocost—the production analogy to indifference curve and budget constraint—we can find the solution to both optimization problems.

Common Procedures:

1. Get isoquant's slope

$$MP_K = \frac{\partial F(K,L)}{\partial K} \quad \text{and} \quad MP_L = \frac{\partial F(K,L)}{\partial L}$$

$$\text{slope of isoquant} = -\frac{MP_K}{MP_L}$$

2. Get isocost's slope

$$\text{slope of isocost} = -\frac{r}{w}$$

3. Slope of isoquant = slope of isocost

$$-\frac{MP_K}{MP_L} = -\frac{r}{w}$$

   and either solve *L* as a function of *K* or *K* as a function of *L*. This gives us the optimal proportion of *K* and *L*.

4. The only difference comes in step 4,

   i. *Product Maximization*

   Substitute the result from step 3 into the cost constraint $rK + wL = \overline{C}$; this gives us the optimal quantities of *K* and *L*. Plugging these into the production function $F(K,L)$ gives us the maximized production.

   ii. *Cost Minimization*

   Substitute the result from step 3 into the quantity constraint $F(K,L) = \overline{F}$;

   this gives us the optimal quantities of *K* and *L*. Plugging these into the cost function $rK + wL$ gives us the minimized cost.

The only difference between product maximization and cost minimization comes in step 4. Notice that in both cases we substitute the optimal proportion of K and L into the production function and the cost function; the only difference is whether we hold production constant or cost constant. Apparently if we set the maximized production from Production Maximization as $\overline{F}$ and do Cost minimization, the resulting minimized cost should equal to $\overline{C}$. Thus the two optimizations are equivalent—they give the same optimized production—cost combination.

III. Lagrange Multiplier

The method of Lagrange Multiplier is a very general method for finding interior optima; please refer to the coming extra handout for more on Lagrange Multiplier.

Comparing the first order conditions (FOC) in slide 6 and 7 of lecture 11 you should be able to see that the first two FOC in each slide are identical; these two FOC give you the optimal proportion of *K* and *L*, corresponding to step 1 - 3 in the equaling of slope method. Add in the third FOC gives the optimal quantities of *K* and *L*, which one substitutes accordingly into the cost function (Cost Minimization, slide 6) or the production function (Production Maximization, slide 7); this is step 4 in the equaling of slope method.

IV. Example

Production function: $F(K,L) = K^{1/3}L^{2/3}$

Input costs: $r = 1, w = 1$

Constants: $\overline{C} = 20, \overline{F} = 2 \cdot 12^{2/3}$

<u>Product Maximization</u>

1. Get isoquant's slope

$$MP_K = \frac{\partial F(K,L)}{\partial K} = \frac{1}{3}K^{-2/3}L^{2/3}$$

$$MP_L = \frac{\partial F(K,L)}{\partial L} = \frac{1}{2}K^{1/3}L^{-1/3}$$

$$\text{slope of isoquant} = -\frac{MP_K}{MP_L}$$

$$= -\frac{\frac{1}{3}K^{-2/3}L^{2/3}}{\frac{1}{2}K^{1/3}L^{-1/3}}$$

$$= -\frac{2L}{3K}$$

2. Get isocost's slope

$$\text{slope of isocost} = -\frac{r}{w} = -1$$

3. Slope of isoquant = slope of isocost

$$-\frac{MP_K}{MP_L} = -\frac{r}{w}$$

$$-\frac{2L}{3K} = -1$$

$$L = \frac{3}{2}K$$

4. Substitute the above into the cost constraint

$$rK + wL = \overline{C}$$

$$1 \cdot K + 1 \cdot \frac{3}{2}K = 20$$

$$\frac{5}{2}K = 20$$

$$K = 8$$

$$L = \frac{3}{2}K$$

$$= 12$$

5. Substitute K = 8 and L = 12 into the production function

$$F(K, L) = K^{1/3}L^{2/3}$$

$$= 8^{1/3}12^{2/3}$$

$$= 2 \cdot 12^{2/3} = \overline{F}$$

<u>Cost Minimization</u>

Step 1- 3 are identical,

4. Substitute the result from step 4 into the production constraint

$$F(K, L) = 2 \cdot 12^{2/3}$$

$$K^{1/3}L^{2/3} = 2 \cdot 12^{2/3}$$

$$K^{1/3}\left(\frac{3}{2}K\right)^{2/3} = 2 \cdot 12^{2/3}$$

$$\left(\frac{3}{2}\right)^{2/3}K = 2 \cdot 12^{2/3}$$

$$K = 8$$

$$L = \frac{3}{2}K$$

$$= 12$$

5. Substitute K = 8 and L = 12 into the cost function

$$rK + wL = 1 \cdot 8 + 1 \cdot 12 = 20 = \overline{C}$$