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ECON 100A Section 111, 112

16th Section Profit Maximization, Perfect Competition (Version 2)

A producer/seller/firm ultimately aim is neither product maximization nor cost minimization but profit maximization.

I. Basic Setting

Profit Function

Profit is given by the profit function $\pi(q)$.

$$\pi(q) = R(q) - C(q)$$

Where $R(q)$ is total revenue from sales and $C(q)$ is total cost. Notice that the profit function depends only on q—we do not really care about inputs anymore. This should not be surprising since we get the total cost function from cost minimization, a topic we just went over.

Total Cost Function

Generally questions on profit maximization will give you a $C(q)$ to start with. If we use the short run cost function as $C(q)$ we get short run profit, and similarly for long run:

$$\pi^{SR}(q) = R(q) - C^{SR}(q)$$
$$\pi^{LR}(q) = R(q) - C^{LR}(q)$$

Short run and long run costs are crucial in the determination of competitive market equilibrium.

Total Revenue Function

Most dynamic in profit maximization enters from the total revenue function. We shall discuss two basic market settings here: perfect competition and monopoly.

II. Profit Maximization Behavior

Condition 1: *MR = MC*

No matter what market settings we are in, a seller's aim is to maximize the profit function:

$$\max_q \{\pi(q)\}$$

Taking the first order condition,

$$\frac{d\pi(q)}{dq} = 0$$

$$\frac{d}{dq}\left[R(q) - C(q)\right] = 0$$

$$\frac{d}{dq}R(q) - \frac{d}{dq}C(q) = 0$$

$$\frac{d}{dq}R(q) = \frac{d}{dq}C(q)$$

$$\boxed{MR = MC} \qquad (1)$$

The profit maximizing condition is *marginal revenue equals marginal cost*—this should not be surprising to us. Marginal cost we are familiar in deriving; we will consider marginal revenue in a moment.

Condition 2: (Not) Shutting-down Condition

<u>Short Run</u>

Product output q only if

$$\boxed{AR(q) \geq AVC^{SR}(q)} \qquad (2)$$

<u>Long Run</u>

If profit drops below zero a seller is better off shutting down, so a seller would produce output only if

$$\pi^{LR}(q) \geq 0$$

This is often expressed in terms of average revenue and average (total) cost.

$$\boxed{AR(q) \geq AC^{LR}(q)} \qquad (3)$$

$AVC^{SR}(q)$ can be higher or lower than $AC^{LR}(q)$; a seller could temporarily shut down in the short run but reopen in the long run if the profit-maximizing output q satisfies

$$AC^{LR}(q) \leq AR(q) < AVC^{SR}(q)$$

or do the opposite if

$$AVC^{SR}(q) \leq AR(q) < AC^{LR}(q)$$

A. Perfect Competition

This is the situation that we focus on in chapter 8 and 9. Related to profit maximization the punch line is *a seller in a perfectly competitive market takes the price of its output as given*. So total revenue for the seller is

$$R(q) = p \cdot q$$

where p is the *market price*, taken as given by the seller. How the market price is determined we shall deal with in a moment; let us first find marginal revenue,

$$MR(q) = \frac{d}{dq}(p \cdot q) = p$$

Notice that average revenue is also p_m , since

$$AR(q) = \frac{R(q)}{q} = p$$

<u>Individual Supply Curve</u>

Combining the above with condition 1 and 2 we have the *supply curve* for the seller:

- $p = MC$ gives us the profit maximizing q at each level of p , while
- The price level at which the seller will start selling is given by

$$p > AVC^{SR}(q) \qquad \text{in short run}$$
$$p > AC^{LR}(q) \qquad \text{in long run.}$$

<u>Market Supply Curve</u>

The *market supply curve* is the horizontal summation of all individual supply curves. In other words, the market output for a given price is the sum of individual outputs at that price. Let $Q(p)$ be the market output and $q_i(p)$ output of seller *i*. If there are *N* sellers then

$$Q_S(p) = \sum_{i=1}^{N} q_i(p)$$

B. Monopoly

Monopoly, as in its common usage, refers to the situation where there is only one seller in the market. We will discuss monopoly in detail in chapter 10; as this moment the punch line is *a monopolist set the price for its output*. The most basic pricing scheme is to set price to the maximum amount consumers are willing to pay for the output, which is given by the demand curve. So

$$R(q) = p(q) \cdot q$$

where $p(q)$ is the *inverse demand*, which is simply the demand curve expressed in the form of price as a function of quantity.

<u>There is no simply supply curve for monopoly.</u> This is due to the monopolist's ability to set its own price.

III. Perfectly Competitive Market Equilibrium

How is the market price in a perfectly competitive market determined? To tackle that we first have to lay out the assumptions of the model:

A1. Price Taking
A2. Product Homogeneity
A3. Free Entry and Exit with infinitely many potential entrants

A1 gives us the supply curve for each individual seller. A2 tells us that they are selling in the exact same market. Finally, A3 tells us that all sellers earn zero profit in long run equilibrium—because any level of positive profit would induce new entrants to the market, raising total output and thereby reducing market price; this process stops only when profit drops to zero.[1] To make the math easy we usually assume the follow too

A4. Sellers have identical cost functions

A. Short Run

1. Short run cost functions
2. Number of sellers is fixed

Short run has dual implications here—except that for cost it also means the number of sellers is fixed. Fixing the number of sellers at N, we have two unknowns p, Q and two equations:

$$\begin{cases} Q_D(p) & \text{Market Demand} \\ Q_S(p) = N \cdot q(p) & \text{Market Supply} \end{cases}$$

Solving $Q_D(p) = Q_S(p)$ gives us the short run equilibrium price and quantity.

B. Long Run

1. Long run cost functions
2. Number of sellers is flexible

In the long run the number of firms can be anything. Remember assumption A3 tells us that in the long run sellers should earn zero profit. Now long run profit for an individual seller is

[1] Technically the process stops when the marginal seller—the seller in production that has the highest cost—earns zero profit. This distinction is important if we allow the sellers to have different costs. Any more profitable sellers would have entered already and could earn positive profit, even in the long run.

$$\begin{aligned}\pi^{LR}(p) &= R(q) - C^{LR}(q)\\ &= AR(q)\cdot q - AC^{LR}(q)\cdot q\\ &= \left[AR(q) - AC^{LR}(q)\right]\cdot q\\ &= \left[p - AC^{LR}(q)\right]\cdot q\end{aligned}$$

The trick is to solve for the q such that $AC^{LR}(q)$ is minimized. This is given by

$$\frac{d}{dq}AC^{LR}(q) = 0 \quad \text{or} \quad AC^{LR}(q) = MC^{LR}(q)$$

This gives us the value of long run equilibrium individual supply q^* and minimized long run average cost $AC^{LR}(q^*)$. We then pick p so that $\pi(p) = 0$. There are three possible cases here:

i. If $AC^{LR}(q)$ is U-shaped

$$p^* = AC^{LR}(q^*)$$
$$N = \frac{Q_D(p^*)}{q^*}$$
$$Q_S = N\cdot q^* = Q_D(p^*)$$

Remember N is the number of firms.

ii. If $AC^{LR}(q)$ is strictly upward sloping

$$p^* = AC^{LR}(0)$$
$$N = \infty$$
$$Q_S = Q_D(p^*)$$

iii. If $AC^{LR}(q)$ is strictly downward sloping

$$N = 1$$
$$\begin{cases} Q_S = q(p) \\ Q_D(p) \end{cases}$$

When there is strictly downward-sloping long run average cost it is most efficient for one single seller to dominate the market, which is monopoly. It is hard to justify the competitive market assumptions in this case.

Note that the long run market supply always equals to one single value, so *it is a horizontal line in all cases*. LR-equilibrium price is always the same; any shift in demand is compensated by the number of sellers in the long run. This is true unless we assume that long run costs depend on the number of firms *N*.

IV. Example

Consider the market for ice cream cones in a small town. There are many potential ice cream vendors. Demand for ice cream cones is given by $P(Q) = 90 - 2Q$.

(a) Suppose all the potential ice cream vendors have identical long run cost functions given by

$$C^{LR}(q) = 20 + 6q + 5q^2$$

Find the long-run equilibrium price, market quantity, number of firms and the amount produced by each firm.

(b) Suppose short run cost when deviating from a long run output q^{LR} is

$$C^{SR}(q) = 20 + 6q + 5q^2 - [q - C^{LR}(q^{LR})]^2$$

Write down the short run cost function for the long run equilibrium in part (a).

(c) Suppose demand surged to $P(Q) = 120 - 2Q$. Find the short run equilibrium price, market quantity, number of firms and amount produced by each firm.

(d) Find the long run equilibrium price, market quantity, number of firms and amount produced by each firm for the demand in part (d).