University of California, Berkeley — Fall 2006
ECON 100A Section 111, 112

21^{th} Section Oligopoly (Version 2)

Oligopoly refers to the situation where the number of sellers in the market is small; more crucially the sellers can affect price either by setting it directly or through output produced. In this class we will focus our attention on the special case of duopoly, the situation where are two sellers.

In all the following cases let the inverse demand be $P_D(Q) = a - bQ$, where $Q = q_1 + q_2$ is the total output. We shall assume that the sellers have identical marginal cost = *c*.

I. Cournot

Each seller chooses its output taking opponent's output as giving. Price determined by (inverse) demand.

Quantity:	$q_1^* = q_2^* = \dfrac{a-c}{3b}$
Price:	$p^* = P_D(q_1^* + q_2^*)$

Output in a Cournot setting is lower than output in a perfectly competitive market but higher than monopoly.
Price is lower than monopoly but higher than competitive market
Total Profit is lower than monopoly but higher than competitive market, which gives zero profit

II. Bertrand

Each seller chooses its price taking opponent's price as giving. Quantity determined by demand. Here we have a price war, resulting in perfectly competitive price.

Quantity:	Q^* given by $P_D(Q^*) = c$ Ant combination of $q_1 + q_2 = Q^*$ is solution
Price:	$p^* = c$

Output and **Price** are the same as competitive market.
Profit is zero as in competitive market.

III. Stackelberg

One seller gets to picks its quantity before the other. The main idea is that the seller who moves first has an advantage—it can take into consideration the other seller's response into consideration. The seller who moves later acts identically to a Cournot competitor; it takes the output of the first player as given when it gets to choose its own output.

Quantity::	$q_1^* = \frac{a-c}{2b}$ $q_2^* = \frac{a-c}{4b}$
Price:	$p^* = P_D(q_1^* + q_2^*)$

Individual Output for Seller 1 is higher than Cournot and lower for Seller 2.
Total Output is higher than Cournot and monopoly but lower than competitive market.
Price is lower than Cournot and monopoly but higher than competitive market
Individual Profit is higher than Cournot for Seller 1 but lower for Seller 2.
Total Profit is lower than Cournot and monopoly but higher than competitive market.

IV. Detailed example

Market Setting

$$P_D(Q) = 30 - 2Q$$
$$c = 4$$

Cournot

Seller 1 solves

$$= \max_{q_1}\{\pi_1\}$$
$$= \max_{q_1}\{P_D(q_1 + q_2)\cdot q_1 - c\cdot q_1\}$$
$$= \max_{q_1}\{[30 - 2(q_1 + q_2)]\cdot q_1 - 4\cdot q_1\}$$

First order conditions:

$$\frac{d\pi_1}{dq_1} = 0$$
$$\frac{d}{dq}\{[30 - 2(q_1 + q_2)]\cdot q_1 - 4\cdot q_1\} = 0$$
$$30 - 4q_1 - 2q_2 - 4 = 0$$
$$q_1 = \frac{30 - 2q_2 - 4}{4}$$

Since the two sellers are identical in all ways, we have

$$q_2 = \frac{30 - 2q_1 - 4}{4}$$

Substituting the formula for q_2 into q_1 ,

$$q_1 = \frac{1}{4}\left[30 - 2\left(\frac{30 - 2q_1 - 4}{4}\right) - 4\right]$$

$$4q_1 = 30 - 15 + q_1 + 2 - 4$$

$$q_1 = \frac{13}{3}$$

Since the sellers are identical we have $q_2 = \frac{13}{3}$

Verify that the formula on the first page is correct: $\frac{a-c}{3b} = \frac{30-4}{3 \cdot 2} = \frac{13}{3}$

Price: $P_D(q_1 + q_2) = 30 - 2(q_1 + q_2) = 30 - 2\left(\frac{13}{3} + \frac{13}{3}\right) = \frac{38}{3}$

Bertrand

$$P_D(Q) = c$$

$$30 - 2Q = 4$$

$$Q = 13$$

so $q_1 + q_2 = Q = 13$

Price equals marginal cost = 4

Stackelberg

Always solve *first* the seller that moves *last*—this method is called *backward induction*. Seller 2's choice is exactly the same as in the Cournot case since q_1 has already been decided by Seller 1 when Seller 2 gets to choose. So from the Cournot case we know

$$q_2 = \frac{30 - 2q_1 - 4}{4}$$

Taking this into account, Seller 1 solves

$$= \max_{q_1}\{\pi_1\}$$

$$= \max_{q_1}\{P_D(q_1 + q_2) \cdot q_1 - c \cdot q_1\}$$

$$= \max_{q_1}\left\{\left[30 - 2\left(q_1 + \frac{30 - 2q_1 - 4}{4}\right)\right] \cdot q_1 - 4 \cdot q_1\right\}$$

First order condition:

$$\frac{d}{dq_1}\left\{\left[30-2\left(q_1+\frac{30-2q_1-4}{4}\right)\right]\cdot q_1-4\cdot q_1\right\}=0$$

$$30-4q_1-\frac{30}{2}+\frac{4}{2}q_1+\frac{4}{2}-4=0$$

$$q_1=\frac{30-30/2-2}{2}$$

$$=\frac{13}{2}$$

Substitute this into the formula for q_2 gives $q_2=\frac{30-2(13/2)-4}{4}=\frac{13}{4}$

Which are exactly what the formulas give us.

Price is $P_D(q_1+q_2)=30-2(q_1+q_2)=30-\left(\frac{13}{2}+\frac{13}{4}\right)=\frac{81}{4}$