University of California, Berkeley Fall 2006
ECON 100A Section 111, 112

<u>22th Section Game Theory (Version 2)</u>

I. Pure Strategy Nash Equilibrium (PSNE)

Each player chooses a single *action*. Given what action the other player has chosen, each player has no incentive to change her action.

To find PSNE(s), on a game matrix circle each player's best action for each of opponent's possible action. Any cell with two circles is a PSNE.

Example

		Player 2: Action C	Player 2: Action D
Player 1	Action A	10,10	16,9
	Action B	9,9	12,12

First circle the best action for Player A.

If Player 2 chooses Action C,

		Player 2: Action C	
Player 1	Action A	(10),10	
	Action B	9,9	

If Player 2 chooses Action D,

			Player 2: Action D
Player 1	Action A		(16),9
	Action B		12,12

Do the same for Player 2 we get

		Player 2: Action C	Player 2: Action D
Player 1	Action A	(10),(10)	(16),9
	Action B	9,9	12,(12)

So (Action A, Action C) is the only PSNE in this case. Note that PSNE is not necessarily efficient—both players are better of with (Action B, Action D); that pair is unsustainable however because Player 1 will have incentive to deviate to Action A for even higher payoff.

II. Mixed Strategy Nash Equilibrium (MSNE)

Each player chooses a *probability* to randomize among different actions. Given the probabilities the other player has chosen, each player has no incentive to change her probability.

Finding MSNE(s)

A general game structure: Player 2

		Action C	Action D
Player 1	Action A	$1_{AC}, 2_{AC}$	$1_{AD}, 2_{AD}$
	Action B	$1_{BC}, 2_{BC}$	$1_{BD}, 2_{BD}$

Assume Player 1 chooses Action A with probability p, Player 2 chooses Action C with probability q.

1. *Equalize the expected payoffs of Player 1 from taking Action A and from taking Action B to find q.*

Expected payoff of Player from taking Action A = $q \cdot 1_{AC} + (1-q) \cdot 1_{AD}$

Expected payoff of Player from taking Action B = $q \cdot 1_{BC} + (1-q) \cdot 1_{BD}$

Equaling the two gives

$$q \cdot 1_{AC} + (1-q) \cdot 1_{AD} = q \cdot 1_{BC} + (1-q) \cdot 1_{BD}$$

Rearrange to get q.

2. *Equalize the expected payoffs of Player 2 from taking Action C and from taking Action D to find p.*

Expected payoff of Player from taking Action C = $p \cdot 2_{AC} + (1-p) \cdot 2_{BC}$

Expected payoff of Player from taking Action D = $p \cdot 2_{AD} + (1-p) \cdot 2_{BD}$

Equaling the two gives

$$p \cdot 2_{AC} + (1-p) \cdot 2_{BC} = p \cdot 2_{AD} + (1-p) \cdot 2_{BD}$$

Rearrange to get p.

Example

Player 2

		Action C	Action D
Player 1	Action A	1,-1	2,2
	Action B	0,2	3,1

You can verify that there is no PSNE in this game. Let us find the MSNE.

Equate the expected payoff of Player 1

$$q \cdot 1 + (1-q) \cdot 2 = q \cdot 0 + (1-q) \cdot 3$$
$$2 - q = 3 - 3q$$
$$q = \frac{1}{2}$$

Equate the expected payoff of Player 2

$$p \cdot (-1) + (1-p) \cdot 2 = p \cdot 2 + (1-p) \cdot 1$$
$$2 - 3p = 1 + p$$
$$p = \frac{1}{4}$$

So the MSNE is for Player 1 to choose Action A for 1/4 of the time and Player 2 to choose Action C for 1/2 of the time.

III. Dominating Strategy

Dominate/Dominating Strategy	An action that a player always chooses, no matter what the other player does
Dominated Strategy	An action that player never chooses, no matter what the other player does

<u>Finding Dominating Strategy</u>

A row with all of Player 1's circles is a dominating strategy for Player 1

A column with all of Player 2's circles is a dominating strategy for Player 2

<u>Finding Dominated Strategy</u>

A row with no Player 1's circles is a dominated strategy for Player 1

A column with no Player 2's circles is a dominated strategy for Player 2

<u>Example</u>

		Player 2		
		Action C	Action D	Action E
Player 1	Action A	10,10	16,9	14,9
	Action B	9,9	12,12	7,11

Action A is a dominating strategy for Player 1, because all her circles (the circle over 10 and the circle over 16) are on Action A's row.

Action B is a dominated strategy for Player 2, because none of her circles is on Action B's row.

Player 2 has no dominating strategy because both Action C and Action D have some of his circles.

Action E is a dominated strategy for Player 2 because it has none of Player 2's circles.