University of California, Berkeley Fall 2006
ECON 100A Section 111, 112

5th Section Finding the Optimal Allocation with IC and BC (Version 2.2)

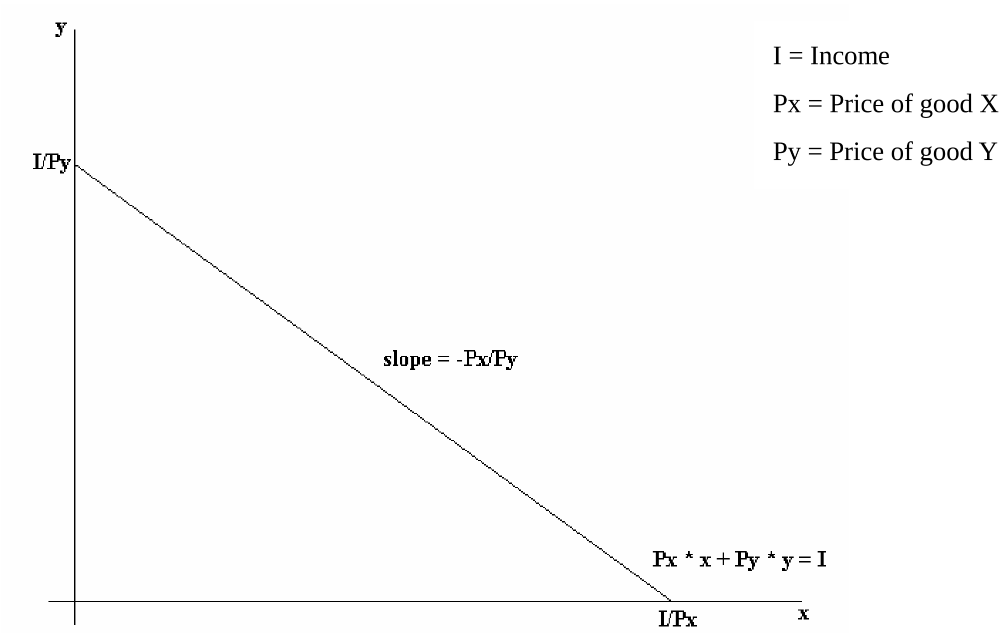
The whole point of having indifference curve (IC) and budget constraint (BC) is to determine the *optimal allocation*—the feasible bundle that gives the highest utility to the individual. By now you should be very familiar with where the optimal allocation is graphically; in this section we shall work it out mathematically.

I. Basics

A problem set or exam question that involves solving out the optimal allocation mathematically would certainly have very simple functional forms; nothing weird like what we have seen last section.

A. Budget Constraint

What you need to familiar yourself with is the basic linear BC. I have summarized the important information below:



With the more-is-better assumption the line is the only thing that matters; on any point on the line the individual is spending all her available income. The most important number in terms of solving for the optimal allocation is the slope, which represents the *relative price* of the two goods in concern. Relative price simply means the number of oranges one needs to give up in order to get an additional apple. Remember that slope =

dy/dx = -Px/Py, so -Px/Py is the *number of Y one gets from getting an additional unit of X*; not surprisingly this is negative.

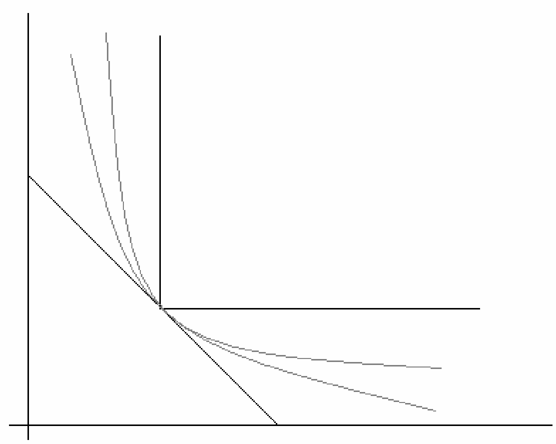
B. Indifference Curve

Again what matters is the slope. The negative of the slope has a fancy name *marginal rate of substitution* (MRS). Remember that points on an indifference curve give the same utility; MRS thus means *upon getting an additional unit of X, the number of Y needed to be taken away in order to give the individual the same level of utility as before*. If more is better MRS is positive.

Diminishing Marginal Rate of Substitution (DMRS)

Mathematically DMRS means the slope is *increasing* (remember that the slope is negative). Economically it means that the individual prefers some of everything than extremes; it follows that with DMRS the average of two given bundles is preferred to either of the bundles. In terms of solving for the optimal allocation it guarantees the uniqueness of the optimal allocation—i.e. you get only one solution.

Any indifference curves with DMRS are bounded between one of perfect substitution and one of perfect complements.



C. Solution Types

Internal solution—individual buys some of everything

Corner solution—individual buys only some of the goods

At first sight internal solutions seem pretty reasonable, but for a little longer you would realize that most of us purchase less than a faction of a percent of all goods that is available. Nevertheless economists like internal solutions because of its mathematical elegancy—MB = MC only works with internal solutions.

II. Step-by-Step

A. Method 1: Equating Slopes

i. *Get BC's slope*

This is usually an actual number.

ii. *Get IC's slope*

This should be a formula of x and y except in the perfect substitution case. A tip: Finding dy/dx by dividing -dU/dx over dU/dy is often easier than getting the slope dy/dx directly.

iii. *Substitution BC's slope into IC's slope and solve Y as a function of X*

iv. *Substitution the above function into BC and solve for x*

v. *Solve for y by substituting the value of x into IC or BC*

B. Method 2: Lagrangian Multiplier

i. *Setup the Lagrangian*

"Lagrangian" is just a fancy name for the following equation:

$$L = U(x, y) + \lambda \cdot BC'$$

where λ is a unknown constant and

$$BC' = P_x \cdot x + P_y \cdot y - I$$

ii. *Take partial derivatives of the Lagrangian with respect to x and y and equate them to zero*

That is, find

$$\frac{\partial L}{\partial x} = 0 \quad \text{and} \quad \frac{\partial L}{\partial y} = 0$$

iii. *Solve the systems of equations consisting of the partial derivatives from above and BC*

Remember the formula for BC is $P_x \cdot x + P_y \cdot y = I$

C. Example

The following problems deal with Jaylum's preferences for music downloads (m) and pizza (p), which are given by the following utility function:

U(m, p) = mp

(a) Find the equation for Jaylum's Marginal Rate of Substitution of m in terms of p.

(b) Assume Jaylum faces prices of $1 downloads and $2 pizza slices, what is the value of the marginal rate of substitution at Jaylum's optimal choice? Why?

(c) If Jaylum has $60 to spend, write an equation representing Jaylum's budget constraint.

(d) What is the optimal bundle of goods Jaylum purchases?

Method 1: Equating Slopes

i. Get BC's slope

$P_p = 1 \qquad P_m = 2 \qquad I = 60$ so

BC: $$m + 2p = 60$$
$$m = 60 - 2p$$

Slope of BC = -2 (1)

ii. Get IC's slope

$$\frac{\partial U(m,p)}{\partial p} = m \quad \text{and} \quad \frac{\partial U(m,p)}{\partial m} = p \quad \text{so}$$

slope of IC = $$-\frac{\frac{\partial U(m,p)}{\partial p}}{\frac{\partial U(m,p)}{\partial m}} = -\frac{m}{p} \qquad (2)$$

iii. Slope of BC = slope of IC

$$-\frac{m}{p} = -2 \qquad (3)$$
$$m = 2p$$

iv. Substitute (3) into BC

$$2p = 60 - 2p$$
$$p = 15$$

and so $m = 2p = 30$

Method 2: Lagrangian Multiplier

$$L = mp + \lambda(m + 2p - 60)$$

$$\frac{\partial L}{\partial p} = m + \lambda = 0 \quad \text{and} \quad \frac{\partial L}{\partial m} = p + \lambda \cdot 2 = 0$$

$m = -\lambda$ and $p/2 = -\lambda$

so $m = 2p$

The remaining steps are identical to iv. in method I.

Corner Solution

If the optimal allocation is unsolvable or consist of negative numbers an interior solution does not exist. In this situation you need to find the corner solution. In our setting this means finding

$$U\left(0,\frac{I}{P_y}\right) \text{ and } U\left(\frac{I}{P_x},0\right)$$

These correspond to the utilities from spending all income on only one good. The higher of the two is the corner solution.

Why does this make sense?