University of California, Berkeley Fall 2006

ECON 100A Section 111, 112

9th Section Choice under Uncertainty (Version 2.5)

In the real world there is plenty of uncertainty—investment, accident, gamble are a few examples. How does a rational individual choose under uncertainty? This part of the course attempts to provide a basic framework to answer this question.

I. Basics Probability

Let *i* be some event, $\Pr_i$ be its probability of happening and X_i its payoff.

Expected Value: $E[X] = \sum_i \Pr_i \cdot X_i$

Variance: $Var(X) = E[X^2 - E(X)]$

$= E[X^2] - E(X)^2$

Standard Deviation: $\sigma_x = \sqrt{Var(X)}$

In economics we often describe situations with uncertainty as a bet. Expected value is also called the *mean*. In general $\Pr_i$ and X_i are functions of *i*; in this class the value of $\Pr_i$ is usually given while X_i often needs to be calculated.

II. Expected Utility Theory

In order to compare different choices under uncertainty we need a way to summarize their uncertain payoffs into a certain number, just like how we went from preferences to utility previously. The fundamental theory on this is *expected utility theory* (hereon EUT)

Let *i* be some event, $\Pr_i$ be its probability of happening and X_i its payoff. The *expected utility* is defined as:

$$E[U(X)] = \sum_i \Pr_i \cdot U(X_i)$$

Suppose facing with two situations with uncertainty with payoffs X_i and Y_i. EUT states that the individual would prefer the first to the second if

$$E[U(X)] > E[U(Y)]$$

In this class you are basically going to be asked to do just that.

III. Risk Attitudes

i. Risk Neutral

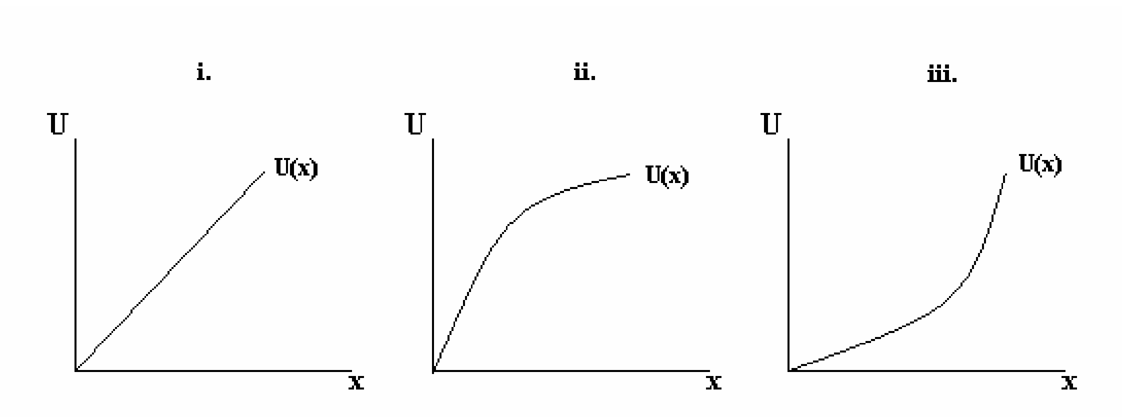
- Benchmark “rational” case; usual assumption on firms
- Equals to linear utility function—Constant MU
  $\alpha U(x_1) + (1-\alpha)U(x_2) = U(\alpha x_1 + (1-\alpha)x_2)$
- $E[U(X)] = U(\Pr_{x_1}\ x_1 + \Pr_{x_2}\ x_2)$

ii. Risk Averse

- Usual assumption on consumers
- Equals to concave utility function—Diminishing MU
  $\alpha U(x_1) + (1-\alpha)U(x_2) < U(\alpha x_1 + (1-\alpha)x_2)$
- $E[U(X)] < U(\Pr_{x_1}\ x_1 + \Pr_{x_2}\ x_2)$

iii. Risk Loving

- Gamblers, etc.
- Equals to convex utility function—Increasing MU
  $\alpha U(x_1) + (1-\alpha)U(x_2) > U(\alpha x_1 + (1-\alpha)x_2)$
- $E[U(X)] > U(\Pr_{x_1}\ x_1 + \Pr_{x_2}\ x_2)$



The above attitudes represent the earliest attempts from economists to model the different behaviors under risk. While insightful they cannot explain many phenomena, such as the purchase of insurance while gambling.

IV. Risk Premium

Risk Premium is the horizontal difference between the expected utility from a bet and the utility function *at the same utility level*. To find the risk premium,

i. Find the expected utility of the bet and its horizontal position x_1

ii. Find the point on the utility function that gives the same level of utility as the bet—i.e. have the same vertical position. Mark its horizontal position as x_2

iii. Risk Premium is $x_1 - x_2$

V. Examples

1. You are spending time with your friend Juanita (she loves to gamble). You tell her you have 25 on you and she proposes a bet. She'll flip a fair coin and if you call it correctly she'll owe you \$11, but if not you owe her \$10. Your utility for money is given by $U(M) = M^{1/2}$, and Juanita's utility for money is given by $U(M) = M^{5/4}$.

(a) Do you accept the gamble?

(b) What if you would only owe her \$7, would you make the gamble?

(c) At what amount would you pay so that you are indifferent between making the bet and not?

(d) If you were forced to make the bet, how much would you be willing to pay to avoid it?

2. A farmer in Ghana has a plot of land and can plant either Cassava or Pineapples. Given a good year of rain, pineapples are much more profitable than Cassava. However, in a bad year of rain Pineapples do much worse than Cassava. Assume that the farmer has no access to insurance and has to make the planting decision before he knows the outcome of rain. The pay-off matrix and probability of good/bad year of rain is given below.

Table 1: Farmer's Pay-Off Matrix per Plot

	Good Rain Year	Bad Rain Year
Pineapple	121	25
Cassava	64	49
Probability	$^1/_3$	$^2/_3$

The farmer's utility for income is given by: $U(I) = I^{1/2}$.

(a) What is the expected pay-off for each type of crop?

(b) Which does the farmer plant? (Must answer why)

(c) Now, assume that the government would like more farmers to plant Pineapples. They create an insurance mechanism that would ensure the income of Pineapples in a good year. What is the maximum premium they can charge that would induce this farmer to plant Pineapples?

(d) Under the above insurance scheme, what should the government expect to pay?

(e) Now assume the farmer's neighbor invented a device that could perfectly predict the weather, thus giving the farmer perfect information on what weather to expect. Up to what amount would the farmer be willing to pay for the device, i.e. what is the value of information?