Dominant Firm Example

In sections I used a different fringe supply equation then what I had in mind when I drew the graphs. Instead of $Q_S(P) = P + 2$ what I had in mind was $Q_S(P) = P - 2$. The steps I took in doing the math is correct but it does not match up with the graphs I drew.

What I had in mind:

Demand:	$Q_D(P) = 10 - P$
Fringe Supply:	$Q_S(P) = P - 2$
MC of Dominant firm:	$MC(Q) = 1$

Step 1: Find the Residual Demand

Residual demand equals to the difference between market demand and fringe supply when price is
- Below the intersection of market demand and fringe supply[1], and
- Above the vertical intercept of fringe supply

Otherwise residual demand equals to market demand.

Intersection of Market Demand and Fringe Supply:
$$Q_D(P) = Q_S(P)$$
$$10 - P = P - 2$$
$$P = 6$$

Vertical Intercept of Fringe Supply :
$$Q_S(P) = 0$$
$$P - 2 = 0$$
$$P = 2$$

$$\text{Residual Demand between 2 and } 10 = Q_D(P) - Q_S(P)$$
$$= (10 - P) - (P - 2)$$
$$= 12 - 2P$$

So the final residual demand is

$$\text{Residual Demand} = \begin{cases} 12 - 2P & \text{if } 2 \leq P < 6 \\ 10 - P & \text{otherwise} \end{cases}$$

---

[1] In sections I said "below the vertical intercept of market demand"; that works too since residual demand already reaches zero by then.

<u>Step 2: Find Dominant Firm's Optimal Quantity</u>
Optimal quantity is given by $MR = MC$. MR, however, is not inverse market demand with double the slope but inverse *residual* demand with double the slope.

We get inverse residual demand by inverting residual demand,
$$Q = 12 - 2P$$
$$P = 6 - Q/2$$

$$Q = 10 - P$$
$$P = 10 - Q$$

We also need to invert the range,
$$P = 2$$
$$Q = 12 - 2(2) = 8$$

$$P = 6$$
$$Q = 12 - 2(6) = 0$$

So
$$\text{Inverse Residual Demand} = \begin{cases} 6 - Q/2 & \text{if } 0 < Q \le 8 \\ 10 - Q & \text{otherwise} \end{cases}$$

MR is inverse residual demand with double the slope,

$$MR = \begin{cases} 6 - Q & \text{if } 0 < Q \le 8 \\ 10 - 2Q & \text{otherwise} \end{cases}$$

Note that for $Q > 8$, $MR = 10 - 2Q < $ -6; the dominant firm would never produce at this range. Thus the first case of MR is what we are going to use for $MR = MC$,
$$MR = MC$$
$$6 - Q = 1$$
$$Q = 5$$
The dominant firm would produce 5 units of output.

<u>Step 3: Find Price and Fringe Quantity Supply</u>
Price is given by substituting dominant firm's output into the inverse residual demand.
Since Q = 5 is between 0 and 8 we use the first case of the inverse residual demand.

$$P = 6 - Q/2$$
$$= 6 - 5/2$$
$$= 3.5$$

Fringe quantity supply is given by the fringe supply curve at the price we just found,

$$Q_S(3.5) = 3.5 - 2$$
$$= 1.5$$

Adding in dominant firm's output we have the total output,

$$\text{Total Output} = 5 + 1.5$$
$$= 6.5$$

Finally we can verify that the total output gives us a price of 3.5 at the total output quantity we just found,

$$Q_D = 10 - P$$
$$6.5 = 10 - P$$
$$P = 3.5$$

P

10

Q_D

Q_S

Residual
Demand

6

3.5

1

1.5   5   8   10

Q

MR

MR